PART-I (1 Mark)

MATHEMATICS

1. The coefficient of x^4 in the expansion of $(x^{1/2} - x^{2/3})^7$ is

   A 0 **B** -7 **C** 35 **D** -35

2. The number of complex numbers z such that $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$ and $|z| = 1$ is

   A 2 **B** 4 **C** 6 **D** 8

3. Let A be an invertible 2×2 real matrix. If $A^{-1} = \begin{bmatrix} 35 & 37 \\ 41 & 43 \end{bmatrix}$ then $\det(12A)$ equals

   A -1728 **B** -1 **C** -12 **D** $-1/12$

4. Given $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$, the value of

   $$\sum_{n=1}^{\infty} \frac{1 + 3 + 5 + \cdots + (2n-1)}{1^3 + 2^3 + 3^3 + \cdots + n^3}$$

   is

   A $2\pi^2/3$ **B** $4\pi^2/3$ **C** $4(\frac{\pi^2}{6} - 1)$ **D** $\pi^2/6$

5. If the slope of one of the lines represented by $4ax^2 + xy + 4y^2 = 0$ is the square of the other, then a equals

   A 1/8 **B** 1/4 **C** -1/4 **D** -1/8

6. Let A and B be two points on the parabola $y = 2x^2 + x - 2$ such that the origin is the midpoint of the line segment joining A to B. The length of AB is

   A 2 **B** 3 **C** $2\sqrt{3}$ **D** $2\sqrt{2}$

7. The triangle formed by x-axis, y-axis and the line $3x + 4y + c = 0$ has inradius 1. Then the value of $|c|$ is

   A 12 **B** 7 **C** 5 **D** 1

8. Let p, q and r denote the lengths of the sides QR, PR and PQ of a triangle PQR respectively. Then $p\cos^2(R/2) + r\cos^2(P/2)$

   A equals q

   B equals $\frac{p+q+r}{2}$


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C equals $\dfrac{p+q+r}{4}$

D cannot be determined with the given data

9. The number of solutions of the equation

$$3\cos^2 x \sin^2 x - \sin^4 x - \cos^2 x = 0$$

in the interval $[0, 2\pi]$ is

A 8 **B** 4 **C** 6 **D** 3

10. If $\theta = \dfrac{1}{2}\sin^{-1}(1/4)$ then

$$64\sin\theta + 64\cos\theta - 8\sec\theta - 8\text{cosec}\theta + \tan\theta + \cot\theta$$

equals

A 8 **B** 4 **C** 0 **D** -1

11. Let

$$L = \lim_{n\to\infty}\left\{\lim_{x\to 0}\left(x\sin\left(\frac{1}{x}\right) + x^2\sin\left(\frac{1}{x^2}\right) + \cdots + x^n\sin\left(\frac{1}{x^n}\right)\right)\right\}.$$

Then

A L does not exist **B** $L = 1$ **C** $L = 0$ **D** $L = -1$

12. Let $f(x)$ be a function defined on $[0, 2]$ by

$$f(x) = \begin{cases} x^3 & \text{for } x \geq 1 \\ ax^2 + bx + c & \text{for } x < 1 \end{cases}$$

where a, b and c are constants such that $f(x)$ has second derivative at $x = 1$. Then a equals

A -6 **B** -3 **C** 6 **D** 3

13. If the function $f(x) = x^3 - 3ax^2 + b$ is strictly increasing for $x > 0$, then which of the following is always true?

A a can take any real value **B** $a \leq 0$ **C** $a < 0$ **D** $a \geq 0$

14. The value of the integral

$$\int_1^3 [x]\cos\left(\pi/2(x - [x])\right)dx,$$

where $[x]$ denotes the largest integer not exceeding x, is

A 6 **B** 1/6 **C** $\pi/6$ **D** $6/\pi$

15. If $f(n) = \dfrac{1}{n}\{(2n+1)(2n+2)\cdots(2n+n)\}^{\frac{1}{n}}$, then $\lim_{n\to\infty} f(n)$ equals

A $4/e$ **B** $27/4e$ **C** $27e/4$ **D** $4e$

16. If $\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3$ are unit vectors such that $\mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3 = \mathbf{0}$ then $|\mathbf{V}_1 - \mathbf{V}_2|$

A equals $\sqrt{3}$

B equals $3/2$

C can be any value in the interval $(0, 2)$

D can be any value in the interval $(3/2, \sqrt{3})$

17. A fair coin is tossed five times. If the outcomes are 2 heads and 3 tails (in some order), then what is the probability that the fourth toss is a head?

A $\dfrac{1}{4}$ **B** $\dfrac{2}{5}$ **C** $\dfrac{1}{2}$ **D** $\dfrac{3}{5}$

18. The number of three element subsets of $\{1, 2, \ldots, 10\}$ sum of whose elements is even is

A 450 **B** 570 **C** 900 **D** 1140

19. Suppose x, y are positive real numbers such that $2\log(x-2y) = \log x + \log y$. Then value of $\dfrac{x}{y}$ is

A 1 **B** $\dfrac{3}{2}$ **C** $\dfrac{2}{3}$ **D** 4

20. Let x, y and z be positive integers such that $gcd(x, y, z) = 1; x < y < z$ and $x^2 + y^2 = z^2$. Then which of the following is always true?

A 2 does not divide x

B 2 does not divide $z(x + y)$

C 4 divides $x + y + z$

D 8 does not divide $x + y + z$