# Zonal Informatics Olympiad, 2002–2003

*Instructions to candidates*

1. The duration of the examination is $2\frac{1}{2}$ hours.
2. The question paper carries 75 marks, broken up into five questions of 15 marks each.
3. Attempt all questions. There are no optional questions.
4. Question 3 has negative marking. No other question has negative marking.
5. There is a separate *Answer Sheet*. To get full credit, you *must* write the final answer in the space provided on the Answer Sheet.
6. Write *only* your final answers on the Answer Sheet. Do *not* use the Answer Sheet for rough work. Submit all rough work on separate sheets.

# Zonal Informatics Olympiad, 2002–2003

*Questions*

1. We are given $n$ integers $x_1, x_2, ..., x_n$, where $n$ is even. Suppose we group these $n$ numbers into $n/2$ pairs and add up each of the pairs. The *weight* of this grouping is the maximum of these sums.

   For example, if the input numbers are 5, 7, 8, -2, 6, 4, 5, 2 and if they are paired up as (5,-2), (7,4), (5,6), (2,8) then the sums of the 4 pairs are 3, 11, 11, and 10. The weight is the maximum of $\{3, 11, 11, 10\}$ and is thus 11.

   For each of the following sets of integers, find a way of grouping them into pairs so that the weight is minimized. In your answer, list out the grouping and then indicate the weight.

   (a) 103, 24, 77, 65, 12, 108, 69, 25, 66, 83

   (b) 83, 112, -16, 72, 161, 75, 152, -23, 77, 247

   (c) 19, 81, 2, 41, 61, 59, 28, 69, 76, 88 *(15 marks)*

2. We start with a two digit positive integer and construct a sequence of two digit numbers as follows. Let the current number be $x$. If $2x$ is less than 100, then the next number in the sequence is $2x$. Otherwise the next number in the sequence is $2x - 100$.

   A number is said to be *good* if we can start with the number and get back to the same number later in the sequence. A number that is not good is said to be *bad*.

   For example, 20 is a good number, because the sequence starting with 20 is 20, 40, 80, 60, 20. So, after four steps, we get back to 20. However 10 is bad because starting from 10 we get the sequence 10, 20, 40, 80, 60, 20, . . . in which 10 never reappears.

   What is the common property that is shared by the set of good numbers?

   *(15 marks)*

3. We are passing a sequence of plates from Atul to Zenobia. Each plate has a number painted on it and no two plates have the same number. We have a table in front of us on which we can temporarily store a single stack of plates. At each step, we are allowed to do one of the following:

   - Take a plate from Atul and pass it on immediately to Zenobia.
   - Take a plate from Atul and put it on top of the stack on the table.
   - If there is at least one plate in the stack on the table, take the topmost plate off the stack and pass it on to Zenobia.

   In this process, we can rearrange the plates that Atul gives us before passing them on to Zenobia. For instance, if the sequence of numbers on the plates given to us by Atul is 1,2,3,4, we can pass them onto Zenobia in the sequence 2,4,3,1 as follows:

- Take plate 1 from Atul and start a stack on the table.
- Pass plate 2 directly from Atul to Zenobia.
- Take plate 3 from Atul and stack it on top of plate 1.
- Pass plate 4 directly from Atul to Zenobia.
- Take plate 3 off the stack and pass it to Zenobia.
- Take plate 1 off the stack and pass it to Zenobia.

We say that an input sequence is *compatible* with an output sequence if it is possible to rearrange this input sequence to produce the output sequence. For instance, we just showed that the input sequence 1,2,3,4 is compatible with the output sequence 2,4,3,1.

Consider the following input and output sequences of plates.

| *Input sequences* | *Output sequences* |
|---|---|
| (1) 5,8,10,3,2,9,7,6,4,1 | (A) 6,2,7,4,5,8,3,10,9,4 |
| (2) 10,1,9,6,5,4,2,8,3,7 | (B) 3,9,7,2,10,6,1,4,8,5 |
| (3) 7,6,2,5,3,4,8,10,4,9 | (C) 9,5,6,1,2,4,10,7,3,8 |

Indicate the compatible pairs from the sets {1,2,3} and {A,B,C}. Note that it is possible for one input sequence to be compatible with more than one output sequence and vice versa. Also, there could be input sequences in {1,2,3} that are not compatible with *any* output sequence from {A,B,C} and vice versa. *(Note: If you mark an input-output pair as compatible when it is not, you get negative marks!)*

*(15 marks)*

4. A thief breaks into a shop selling exotic powdered spices (masalas). He has a sack with him in which he can carry away spices weighing upto $W$ kg. For each spice, the thief knows how much of that spice powder is available in the shop and the total value of that spice powder. The problem is for the thief to decide how much of each spice to steal so that he maximizes the value of the spices that he carries away.

   For instance, suppose that the thief can carry away 20 kg (that is, $W = 20$) and there are three spices available—turmeric, cloves and mustard. There are 18 kg of turmeric with a total value of Rs 2400, 10 kg of cloves with a total value of Rs 1500 and 15 kg of mustard with a total value of Rs 1800. We can represent these values by the following table:

   $W = 20$

   | | turmeric | cloves | mustard |
   |---|---|---|---|
   | *amount* | 18 | 10 | 15 |
   | *value* | 2400 | 1500 | 1800 |

   If the thief fills his sack with all 18 kg of turmeric and 2 kg of mustard, he will walk away with spices worth Rs $2400 + (\frac{2}{15} \times 1800) =$ Rs $2400 + 240 =$ Rs 2640.

Here are three situations in which the thief finds himself. Calculate the maximum value he can steal in each case. In your answer, give the breakup of what the thief carries away in his sack and the total value.

(a) $W = 20$

| | A | B | C |
|---|---|---|---|
| *amount* | 15 | 10 | 18 |
| *value* | 1800 | 1500 | 2400 |

(b) $W = 30$

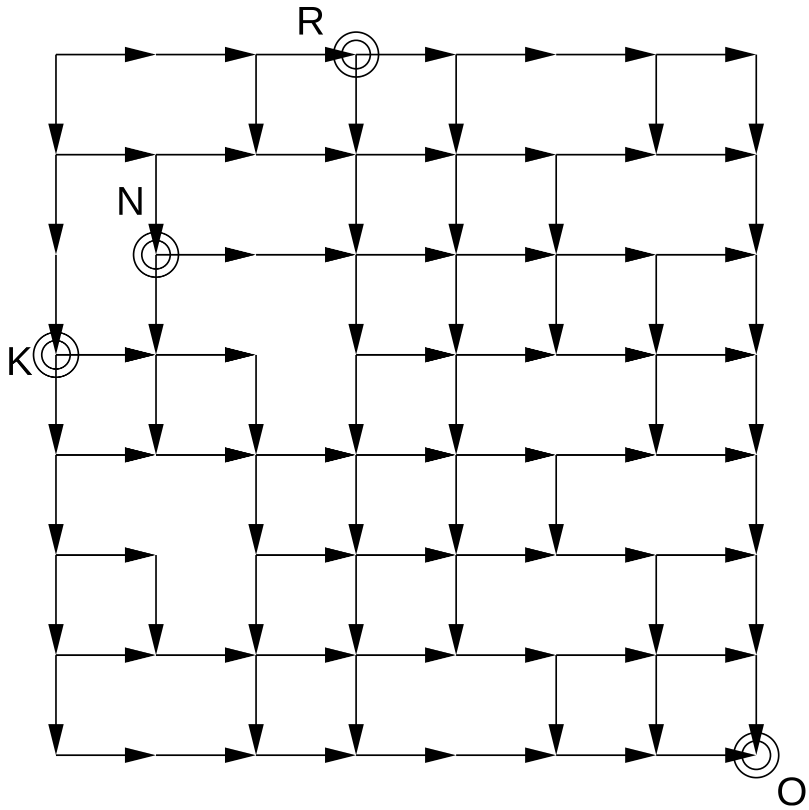
| | A | B | C | D |
|---|---|---|---|---|
| *amount* | 25 | 10 | 15 | 9 |
| *value* | 3000 | 1400 | 1800 | 1200 |

(c) $W = 30$

| | A | B | C | D |
|---|---|---|---|---|
| *amount* | 25 | 10 | 15 | 20 |
| *value* | 2250 | 1100 | 1500 | 1900 |

*(15 marks)*

5. Komal, Narain and Robert all work in the same office. All the roads in the city where they live are one-way and their route from home to office must take this into account. Here is a map of the city. The letters K, N and R mark the homes of Komal, Narain and Robert, respectively, and O marks the office where they work. The direction in which each road can be used is indicated by an arrow. For each person, calculate how many different routes that person can take from home to office.




*(15 marks)*

# Zonal Informatics Olympiad, 2002–2003: *Answer sheet*

| **Roll No:** | **Examination Centre:** |
|---|---|

***Important:*** *Write your final answers (and nothing else) in the space provided.*
*Write all rough work on separate sheets.*

1. (a) Grouping:
   Weight:

   (b) Grouping:
   Weight:

   (c) Grouping:
   Weight:

2.

3. Draw a line connecting each pair of compatible sequences: *(This question has negative marking!)*

| *Inputs* | *Outputs* |
|---|---|
| (1) | (A) |
| (2) | (B) |
| (3) | (C) |

4. (a) Sack contents:
   Total value:

   (b) Sack contents:
   Total value:

   (c) Sack contents:
   Total value:

5. (a) Komal: (b) Narain: (c) Robert:

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| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1. | (a) | (b) | (c) | 2. | | 3. | + | – | Net |
| 4. | (a) | (b) | (c) | 5. | (a) | (b) | (c) | **Total** |

In B, we can check that for each number $c$, all numbers smaller than $c$ that appear after $c$ appear in descending order — in other words, if we have $a < b < c$ and $a, b$ appear after $c$ in B, then $a$ appears after $b$, so the three numbers appear as $c, b, a$, not $c, a, b$. Thus, there is no order violation in B and B is compatible with 1.

This is not the case with C. For instance, we have the sequence 16,11,15, which corresponds to illegally reordering 5,2,9 from sequence 1 as 9,5,2 in sequence C. So, C is not compatible with 1.

Similarly, we can check sequence 2 against B and C.

*2*: 11(10), 12(1), 13(9), 14(6), 15(5), 16(4), 17(2), 18(8), 19(3), 20(7)

*B*: 19(3), 13(9), 20(7), 17(2), 11(10), 14(6), 12(1), 16(4), 18(8), 15(5)

*C*: 13(9), 15(5), 14(6), 12(1), 17(2), 16(4), 11(10), 20(7), 19(3), 18(8)

In B, we have, for instance, the sequence 19,13,17 which corresponds to illegally reordering 9,2,3 from sequence 2 as 3,9,2 in sequence B. So, B is not compatible with 2.

In C, we can check that for each number $c$, all numbers smaller than $c$ that appear after $c$ appear in descending order so there is no order violation and C is compatible with 2.

Finally, we use the same technique to renumber and compare 3 and A.

*3*: 11(7), 12(6), 13(2), 14(5), 15(3), 16(4), 17(8), 18(10), 19(4), 20(9)

*A*: 12(6), 13(2), 11(7), 16(4), 14(5), 17(8), 15(3), 18(10), 20(9), 19(4)

A contains the illegal sequence 16,14,15 corresponding to reordering 5,3,4 from 3 as 4,5,3 so A is not compatible with 3. (Actually, we should also check the renumbering of A where the first 4 in A is renumbered 19 and the second 4 is renumbered 16. In this case, we have the illegal sequence 19,14,15, so the sequences remain incompatible).

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4. (a) Sack contents: 10 kg of B, 10 kg of C
   Total Value: 2833.33

   (b) Sack contents: 10 kg of B, 9 kg of D, 11 kg divided in any way between A and C.
   Total Value: 3920

   (c) Sack contents: 10 kg of B, 15 kg of C, 5 kg of D
   Total Value: 3075

**Justification:**

The thief should choose items according to their value per unit weight. Thus, we compute the value per unit weight of each item and fill the sack starting with the highest value item.

(a) $W = 20$

| | A | B | C |
|---|---|---|---|
| *amount* | 15 | 10 | 18 |
| *value* | 1800 | 1500 | 2400 |
| *unit value* | 120 | 150 | 133.33 |

Therefore, pick items in the order B, C, A. After exhausting all 10 kg of B, add 10 kg of C to fill the sack. Total value is $10 * 150 + 10 * 133.33 = 1500 + 1333.33 = 2833.33$.

(b) $W = 30$

| | A | B | C | D |
|---|---|---|---|---|
| *amount* | 25 | 10 | 15 | 9 |
| *value* | 3000 | 1400 | 1800 | 1200 |
| *unit value* | 120 | 140 | 120 | 133.33 |

Therefore, pick items in the order B, D, and then A or C. After exhausting all 10 kg of B and all 9 kg of D, add 11 kg of A/C to fill the sack. Total value is $10 * 140 + 9 * 133.33 + 11 * 120 = 1400 + 1200 + 1320 = 3920$.

(c) $W = 30$

| | A | B | C | D |
|---|---|---|---|---|
| *amount* | 25 | 10 | 15 | 20 |
| *value* | 2250 | 1100 | 1500 | 1900 |
| *unit value* | 90 | 110 | 100 | 95 |

Therefore, pick items in the order B, C, D, A. After exhausting all 10 kg of B and all 15 kg of C, add 5 kg of D to fill the sack. Total value is $10*110+15*100+5*95 = 1100 + 1500 + 475 = 3075$.

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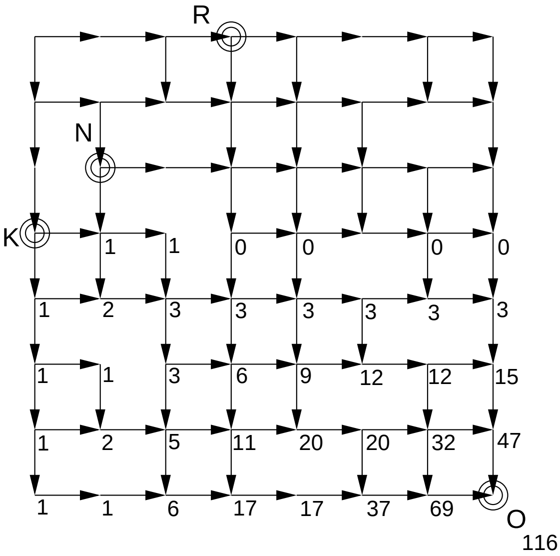
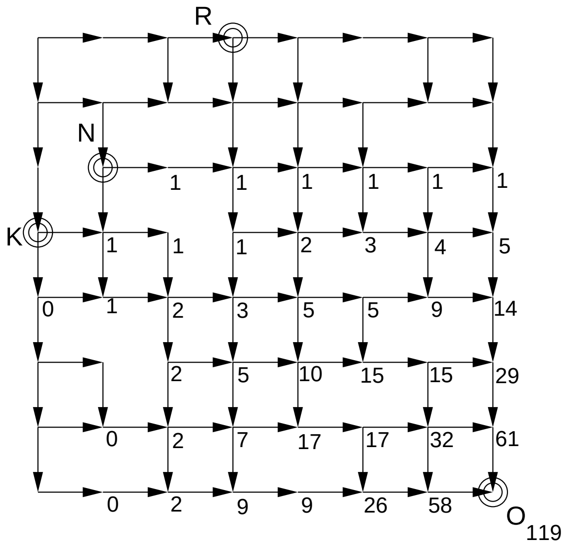
5. (a) Komal: 116
   (b) Narain: 119
   (c) Robert: 97

**Justification:**

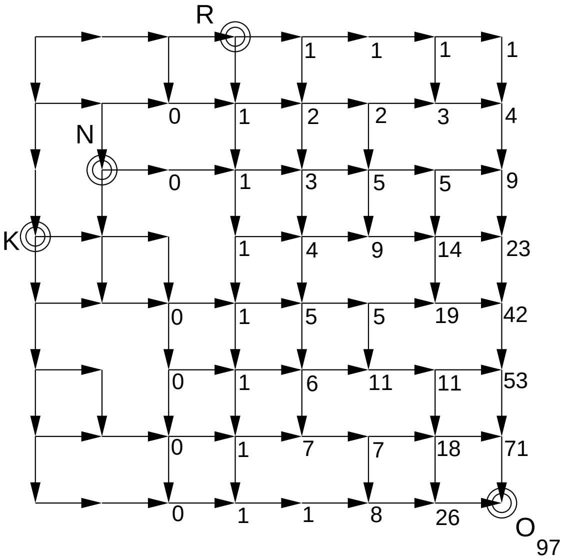
To count the number of ways to reach an intersection, we can add up the number of ways of reaching each intersection that is one step back from the current intersection. Since all the roads in the map are one way pointing down or to the right, each intersection has at most two immediately preceding intersections, above and to the left. We can thus start from the intersections marked K, N and R and systematically fill up row by row the number of ways of reaching each intersection, until we arrive at the figure we want, for the intersection marked O. The computation for each of the three cases is shown below.




Komal




Narain




Robert