

Junior Science

1) (b) We have $x = \frac{F_e}{F_g} = \frac{\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}}{G \frac{m_e^2}{r^2}} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{Gm_e^2}$. By substituting the values of the

constants, we get $x = 4.3 \times 10^{42}$.

2) (a) Refer to the figure. $\angle CBD = \angle CAD = \angle APB - \angle ADB = 4x - 3x = x$. Also $\angle ABD + \angle CBD + \angle CBQ = 180^\circ$. This gives $10x = 180^\circ$ or $x = 18^\circ$. Therefore, $\angle BCD = \angle ACB + \angle ACD = \angle ADB + \angle ABD = 3x + 2x = 5x = 90^\circ$.

3) (a) Using the kinematical equation, $0.4u = u - 9.8(3)$, so that initial velocity $u = 49$ m/s. When the arrow reaches the maximum height, the velocity is zero and hence $0 = 49^2 - 2(9.8)H_{\max}$, giving the expected result.

4) (b)

5) (a) With the usual symbols, we can write for the masses of the two bodies:

$$m_A = \frac{F}{10} \text{ and } m_B = \frac{F}{15}. \text{ Now, } m = m_A + m_B = \frac{F}{10} + \frac{F}{15} = \frac{F}{6} = \frac{F}{a}.$$

This then gives the result.

6) (c)

7) (b)

8) (c)

9) (d) Using the kinematical equation $v^2 = u^2 + 2as$, the velocity with which the ball strikes the ground is 12 m/s (directed downwards). Again the velocity with which the ball rebounds comes out to be 8 m/s (directed upwards). Now, using $u = -12$ m/s, $v = 8$ m/s and $t = 20 \times 10^{-3}$ s, one gets acceleration $a = 1000$ m/s².

10) (b) Obviously, since BD is the bisector of $\angle ABC$, $AD : CD = AB : BC$ and hence the result follows.

11) (c)

12) (a)

13) (a) The kinetic energy $E = \frac{1}{2}mu^2 \Rightarrow u^2 = \frac{2E}{m}$. Again since the final velocity of

each of the buses is zero, we have $0 = u^2 - 2\left(\frac{F}{m}\right)s \Rightarrow s = \frac{mu^2}{2F} = \frac{m\left(\frac{2E}{m}\right)}{2F} = \frac{E}{F}$.

Since, E and F are equal for both the buses, the required ratio is one.

14) (b) The hour hand at 4.30 pm is along a line bisecting second and fourth quadrant.

Therefore, equation of this line is $y = -x$ or $x + y = 0$.

15) (c)

16) (b) Draw yourself an equilateral triangle with each of the sides as $3a$. The area of triangle = $\frac{1}{2}(3a)\left(\frac{\sqrt{3}}{2}3a\right) = \frac{9\sqrt{3}a^2}{4}$. Also, area of the hexagon with side a (which

is the largest that can be fitted inside the triangle) = $6\left(\frac{\sqrt{3}}{4}a^2\right) = \frac{3\sqrt{3}a^2}{2}$. This

area is given to be 320 cm^2 , from which we get $a^2 = \frac{640}{3\sqrt{3}}$ and the result follows.

17) (d) The walls of the prism act as parallel plates and hence there is no deviation and no dispersion.

18) (a) Direction of conventional current is the direction in which positive charge (effectively) moves. Therefore, current in the discharge tube is flowing to the right. Magnitude of current is the net amount of charge moving per second = $2.8 \times 10^{18} e + 1.2 \times 10^{18} e = 4 \times 10^{18} \times 1.6 \times 10^{-19} = 6.4 \times 10^{-1} = 0.64 \text{ A}$

19) (d)

20) (c) For safe operation, maximum power of 40 watt can be dissipated in the series resistance (on right side). Let I be the current in this resistance for 40 watt dissipation in it. Current in each of the two resistances connected in parallel will be $\frac{I}{2}$. Therefore, the total power dissipated in the combination will be sum of the

power dissipated in each of the resistances = $\left(\frac{I}{2}\right)^2 R + \left(\frac{I}{2}\right)^2 R + I^2 R = \frac{3}{2}(I^2 R)$

which on substitution gives $\frac{3}{2}(40) = 60 \text{ watt}$.

21) (d)

22) (d)

23) (b) Let $u = f + x_1$ and $v = f + x_2$. Using mirror formula, $\frac{1}{f} = \frac{1}{f + x_1} + \frac{1}{f + x_2}$ so

that $\frac{1}{f} = \frac{x_1 + x_2 + 2f}{x_1 x_2 + f x_1 + f x_2 + f^2} \Rightarrow f^2 = x_1 x_2 \Rightarrow f = \sqrt{x_1 x_2}$.

24) (b) Using the property of an A. P., we have

$$\frac{c+a-b}{b} - \frac{b+c-a}{a} = \frac{a+b-c}{c} - \frac{c+a-b}{b} \Rightarrow \frac{a^2 - b^2 + ac - bc}{a} = \frac{b^2 - c^2 + ab - ac}{c}$$

$$\Rightarrow \frac{(a-b)(a+b+c)}{a} = \frac{(b-c)(a+b+c)}{c} \Rightarrow ab - ac = ac - bc \Rightarrow b(a+c) = 2ac \text{ and}$$

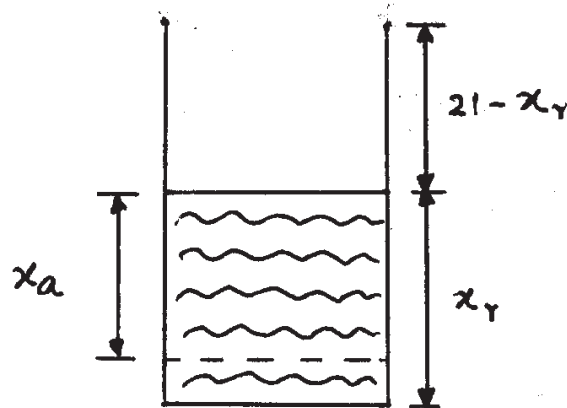
hence the result.

25) (c)

26) (c) Refer to the figure. Let x_r be the real depth and x_a be the apparent depth. The container appears to be half-filled, so that $x_a = 21 - x_r$. But refractive index $n =$

$$\frac{x_r}{x_a} = \frac{4}{3} \Rightarrow x_a = \frac{3}{4}x_r. \text{ Using this in the above relation for } x_a, \text{ we get}$$

$$\frac{7}{4}x_r = 21 \Rightarrow x_r = 12 \text{ cm.}$$



27) (c) Electron moving to the east is equivalent to a current flowing to the west. Using Fleming's left hand rule, we get the direction of magnetic field.

28) (b) Since there is a change of medium at both the surfaces, refraction (that is bending) takes place at each of the two surfaces.

29) (c)

30) (c)

31) (b) The number is divisible by 54, it is clearly divisible by 9. Therefore, the sum of the digits, that is, $(38 + d)$ is divisible by 9. This is possible only if $d = 7$.

32) (c)

33) (a) Actual division gives the remainder $(x + 2)$. Comparing it with the given remainder $(ax + b)$ gives the result.

34) (a)

35) (a)

36) (a)

37) (a) Since $DE \parallel BC$, the sides AB and AC are divided by DE in proportion, that is $AD : DB = AE : EC$. This leads to the expected result.

38) (d)

39) (d)

40) (b) The ray passes from rarer to denser medium and bends away from the normal is not correct.

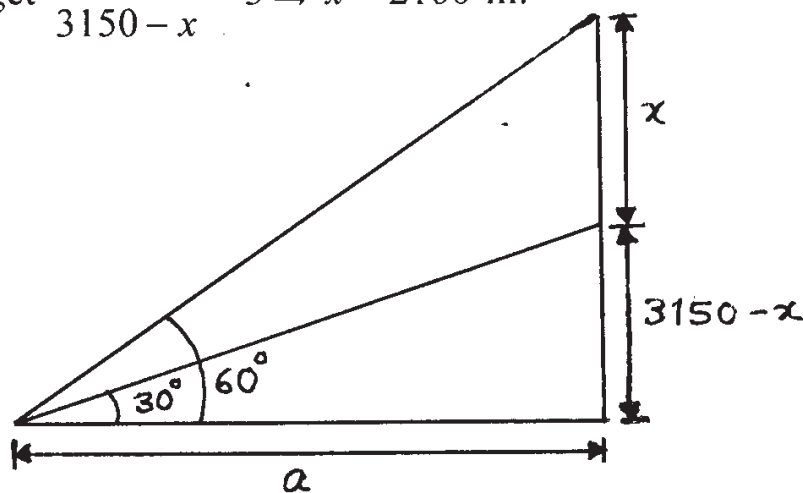
41) (a)

42) (c) Distance from origin is plotted on X axis and change in density of the medium is plotted on Y axis.

43) (c) Use the basic expression: amount of heat lost or gained by a body = mass \times specific heat \times difference in temperature. Thus, we have $Q = x \times c \times t_1 = y \times 1 \times t_2$ where c is the specific heat of the substance.

44) (b) Referring to the diagram, $\tan 30^\circ = \frac{3150 - x}{a} = \frac{1}{\sqrt{3}}$ and $\tan 60^\circ = \frac{3150}{a} = \sqrt{3}$.

Dividing, we get $\frac{3150}{3150 - x} = 3 \Rightarrow x = 2100$ m.



45) (c)

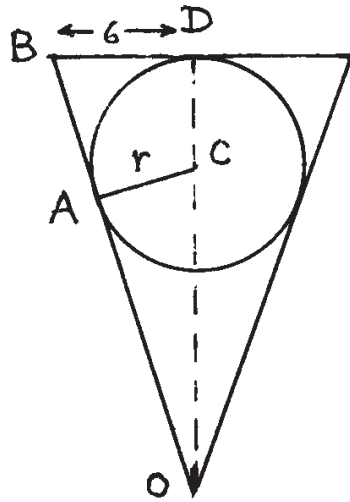
46) (a)

47) (b)

48) (a) We have work done $W = mgh = 0.1 \times 10 \times 10 = 10$ J during upward motion. Note that no work is done by the boy during downward motion of the stone.

49) (b)

50) (a) Refer to the figure.



Triangles BDO and CAO can be easily proved to be similar, so that $AC : OC = BD : OB$. Therefore, $\frac{r}{8-r} = \frac{6}{10} \Rightarrow r = 3$. Hence the required fraction of volume

$$\text{is } \frac{\frac{4}{3}\pi(3)^3}{\frac{1}{3}\pi(6)^2(8)} = \frac{3}{8}.$$

51) (d) Obviously since the satellite moves through vacuum, there is no friction.

52) (a)

53) (b)

54) (a)

55) (a)

56) (c)

57) (d)

58) (d) Note that voltage at point A and as well as at B is 12 volt. In other words points A and B are equipotential and hence no current flows between these points.

59) (d) We have $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}} = \frac{(\sqrt{3} + \sqrt{2})^2}{3 - 2} = 5 + 2\sqrt{6} = 5 + 2(2.449) = 9.898$. Note that

$$\sqrt{600} = 10\sqrt{6} = 24.49, \text{ so that } \sqrt{6} = 2.449.$$

60) (c)

61) (d)

62) (c) Using the property of logarithm, $x^2 - 5x - 65 = 1 \Rightarrow x^2 - 5x - 66 = 0$. This can be factorised to get $x = -6$ or 11 . But since $x < 0$, we have the answer $x = -6$.

63) (b) If initial pressure is p and temperature is T , then final pressure will be $5p$, and

temperature will be $\frac{T}{2}$. Now, use $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ to get the result.

64) (a) Let $2^{2x-1} = a$, so that $2^{1-2x} = \frac{1}{a}$. Using this we get $a + \frac{1}{a} = 2 \Rightarrow (a-1)^2 = 0$.

This then gives $a = 1 \Rightarrow 2^{2x-1} = 1 \Rightarrow 2x - 1 = 0 \Rightarrow x = 0.5$.

65) (c)

66) (c) At point C the magnetic needle remains pointing north when no current flows.

However, a magnetic field stronger than the earth's magnetic field is developed due to the current through the wire. This being directed towards south the deflection of the needle happens to be the maximum.

67) (a) Expand $(a-b)^2$ and similarly $(b-c)^2$, $(c-d)^2$ and $(d-a)^2$. Add them and note that the sum of these squares is ≥ 0 . On simplification we get the result.

68) (c)

69) (c) Add the two equations, we get $(a+b)^2 = 100$, so that $(a+b) = +10$. Note that since a and b are given to be positive, $(a+b) = -10$ is not accepted. This then gives $a = 3.6$ and $b = 6.4$ and hence the answer.

70) (c) Given that the surface areas are equal, $4\pi r^2 = 6a^2 \Rightarrow \frac{r}{a} = \sqrt{\frac{3}{2\pi}}$. With this we

can write the ratio of the volumes as $\frac{\frac{4}{3}\pi r^3}{a^3} = \frac{4}{3}\pi \left(\frac{r}{a}\right)^3 = \frac{4}{3}\pi \left(\sqrt{\frac{3}{2\pi}}\right)^3 = \sqrt{\frac{6}{\pi}}$.

71) (d) If a is the side of the equilateral triangle, its area = $A\sqrt{3} = \frac{\sqrt{3}}{4}a^2 \Rightarrow a^2 = 4A$.

Since the radius of each of the circles is $\frac{a}{2}$, area of each circle = $\pi\left(\frac{a}{2}\right)^2 = \pi A$.

Now, the required area = $A\sqrt{3} - 3\left(\frac{1}{6}\pi A\right) = A\left(\sqrt{3} - \frac{\pi}{2}\right)$.

72) (c) Let x and y be the number of boys and girls respectively, so that $x + y = 1400$.

The average score of the school can be expressed as $\frac{68x + 72y}{x + y} = 69.5$ Solving

this gives $1.5x = 2.5y$ or $3x = 5y$. Solving this simultaneously with $x + y = 1400$ gives the result.

73) (a)

74) (d) Let $a < b$, so that $ac < bc$ or $(ab + ac) < (ab + bc)$ or $a(b + c) < b(a + c)$. This then gives $\frac{a}{b} < \frac{a+c}{b+c} \Rightarrow \frac{a+c}{b+c} > \frac{a}{b}$.

75) (d) When two vectors of equal magnitude include an angle of 120° , their resultant also has the same magnitude as that of each of them. Thus, angle θ must be 60° , twice of which is 120° .

76) (d) Use the formula: area of $\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ where (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are the vertices of a triangle. Substitution gives the area of the triangle to be zero which is option (d)

77) (a)

78) (a)

79) (b)

80) (b) Electric lines of force start from positive charge and terminate on negative charge and are not closed curves.