Indian National Physics Olympiad - 2012
Solutions

Please note that alternate/equivalent solutions may exist. Brief solutions are given below.

1. (a) $\omega_{i+1} = \frac{7}{13}\omega_i + \frac{6}{13}\frac{v}{r}$

   (b) $\omega^* = \frac{v}{r}$

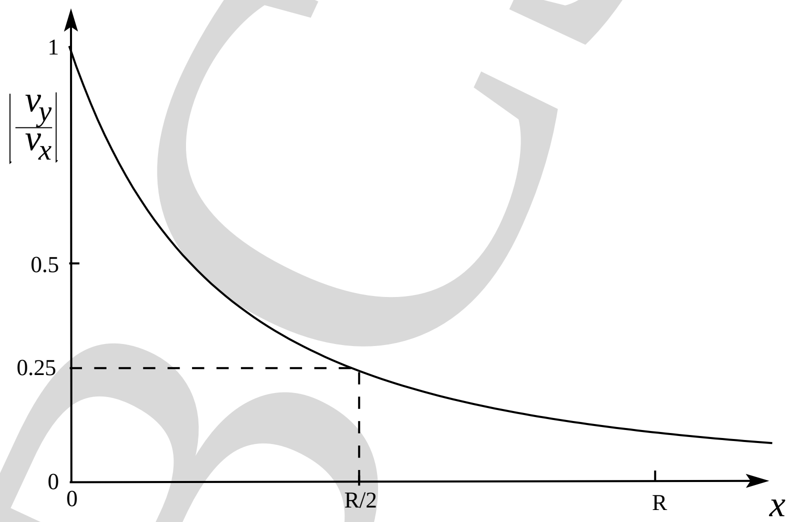
   Argument : Initially ω_i increases until it reaches a value $v = \omega^* r$, i.e. the speed of the falling ball. Thereafter the ball merely "touches" the sphere and does not impart it any momentum.

   (c) $\omega_i = \frac{v}{r}\left(1 - \left(\frac{7}{13}\right)^i\right)$ $\quad\quad$ i=0,1,2,3,......

   (d) $\omega^* = \frac{v}{r}$

2. (a) $v_y = \frac{R^2}{(2x+R)^2} v_x$

   (b) See figure below:



   (c) Speed = 2.22 km$\cdot$ hr^{-1}

3. (a) $\Gamma = \frac{m_a g}{R}\frac{(\gamma - 1)}{\gamma}$

   (b) For $m_a = 29.0$ kg$\cdot$kmol^{-1} ; Γ= approx 10 K$\cdot$km^{-1}

   (c) $\alpha = \frac{\gamma}{\gamma - 1}$

   (d) approximately 30.0 km

   (e) $p_s = p_{s0} \exp\left[\frac{L m_v}{R}\left(\frac{1}{T_{s0}} - \frac{1}{T}\right)\right]$
   where T_{s0} and p_{s0} are the initial points for the integration. A convenient choice would be the triple point of water.

   (f) At z_c atmospheric pressure should be equal to saturation pressure. Condition is

$$p_0 \left(\frac{T_0 - \Gamma z_c}{T_0}\right)^{\gamma/1-\gamma} = p_{s0} \exp\left[\frac{L m_v}{R}\left(\frac{1}{T_{s0}} - \frac{1}{T_0 - \Gamma z_c}\right)\right]$$

4. (a) Magnetic field $= \begin{cases} \dfrac{\mu_0 N I}{l}\hat{k} & \rho < r \\ 0 & \rho > r \end{cases}$

Value of magnetic field $= \begin{cases} 1.26 \times 10^{-2}\ \text{T} & \rho < r \\ 0 & \rho > r \end{cases}$

where ρ is the radial distance.

(b) $L = \dfrac{\mu_0 N^2 \pi r^2}{l}$

Value of $L = 1.97 \times 10^{-2}$ H

(c) $E = 3.95$ J

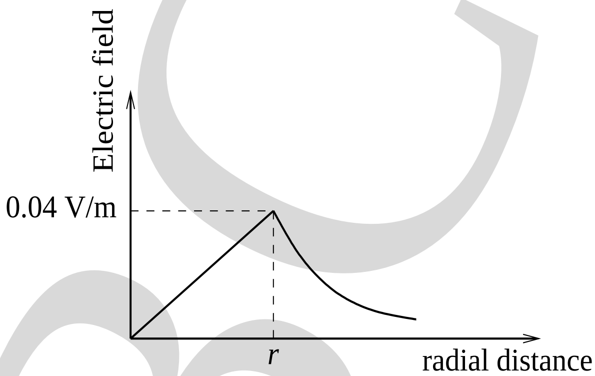
(d) $i = \dfrac{e}{R}(1 - \exp(-Rt/L))$

(e) $e = iR + L\dfrac{di}{dt} - i\dfrac{Lv}{l + vt}$

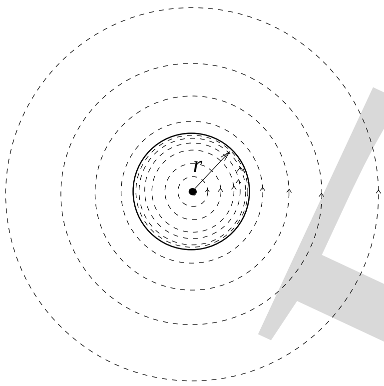
where $L = \mu_0 N^2 \pi r^2/(l + vt)$

(f) Electric field $= \begin{cases} \dfrac{\mu_0 N i_0 \omega \rho}{2l}\sin(\omega t) & \rho < r \\ \dfrac{\mu_0 N i_0 \omega r^2}{2\rho l}\sin(\omega t) & \rho > r \end{cases}$

(g) The plot of E with radial distance:



Lines of forces: Note, the lines of force are dense upto $\rho = r$ and increasingly sparse thereafter.



5. (a) Since $\hbar\omega_0 < E_b$, hence no ionisation by a single photon is possible.

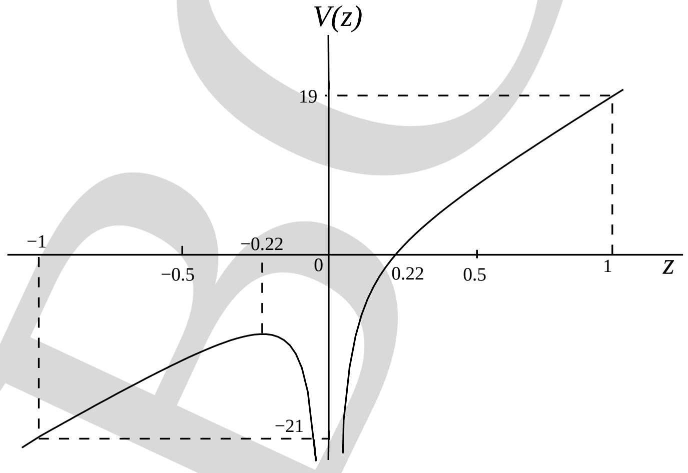
(b) Speed of electron $= \dfrac{eF_0}{m\omega}\sin(\omega t)$

(c) Average kinetic energy $= \dfrac{e^2F_0^2}{4m\omega^2}$

(d) $F_0 = 1.5 \times 10^3$ V$\cdot$ m^{-1}

(e) Potential energy $= -\dfrac{e^2}{4\pi\epsilon_0 r} + eF_0 z$

(f) See figure below:



(g) $F_0 = \dfrac{E^2\pi\epsilon_0}{e^3}$

(h) approx 174 V$\cdot$m^{-1} which is physically possible.

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